

# B5440 – Exercise 1, Review and self-assessment

## Exercises

1. Suppose  $X, X_1, \dots, X_n$  are independent and identically distributed with mean 0 and finite variance but *not* normal. The t-statistic to test the null hypothesis that  $E(X) = 0$  is

$$\frac{\bar{X}_n}{S_n/\sqrt{n}},$$

where  $\bar{X}_n$  is the sample mean and  $S_n$  is the sample standard deviation. Show that the t-statistic converges in distribution to a standard normal and note which named theorems you are using.

2. Suppose  $X \sim \text{exponential}(\theta)$  and  $Y \sim \text{exponential}(\eta)$  with densities  $f_\theta(x) = \theta e^{-\theta x}$ ,  $f_\eta(y) = \eta e^{-\eta y}$ . In the *uncensored* case, we observe both  $X$  and  $Y$ . In the *right censored* case we observe  $(Z, \Delta) = (\min(X, Y), 1\{X \leq Y\})$ .

(a) Densities.

- i. Find the joint density of  $(X, Y)$ .
- ii. Find the joint density of  $(Z, \Delta)$ .

(b) Scores. Find the scores for  $\theta$  and  $\eta$ :

- i. in the uncensored case.
- ii. in the right censored case.

(c) Information. Find the information for  $\theta$ :

- i. in the uncensored case.
- ii. in the right censored case

3. If  $X \geq 0$  and has distribution function  $F$ , show that

$$E(X) = \int_0^\infty (1 - F(x)) dx.$$

## Reading

ABG Chapter 1.