B5440 – Exercise 2, Processes and Martingales

Exercises

1. Answer and prove the following:

a. Let $X_1, X_2, \ldots$ be independent with mean 0. Is $M_n = X_1 + \cdots + X_n$ a martingale?
b. Let $X_1, X_2, \ldots$ be independent with mean μ. Is $S_n = X_1 + \cdots + X_n$ a martingale?
c. Let $X_1, X_2, \ldots$ be independent with mean 1. Is $M_n = X_1 \cdot X_2 \cdots X_n$ a martingale?

2. Show that if M is a martingale then so is the stopped process M^T.

3. Poisson process compensator:

Let $N(t)$ be the number of events in $[0, t]$ where $N(t) - N(s) \sim \text{Poisson}((t-s)\lambda)$ for $s < t$ and N has independent increments.

Does the Doob-Meyer decomposition apply to $N(t)$? If so, identify the compensator of $N(t)$. Also find the predictable variation process.

Reading

ABG Chapter 2.