B5440 – Exercise 4

Exercises

1. Let $N(t)$ be a counting process that satifies the multiplicative intensity model $\lambda(t) = \alpha(t)Y(t)$ and consider the null hypothesis

$$H_0 : \alpha(t) = \alpha_0(t), t \in [0, t_0]$$

where α_0 is a known/fixed function.

a. Let $J(t) = I(Y(t) > 0)$ and $\hat{A}(t) = \int_0^t J(s)/Y(s)\, dN(s)$ and introduce $A_0^*(t) = \int_0^t J(s)\alpha_0(s)\, ds$. Show that $\hat{A} - A_0^*$ is a mean zero martingale under the null.
b. Find an expression for the predictable variation process of $\hat{A} - A_0^*$ under the null.
c. Consider the test statistic

$$Z(t_0) = \int_0^{t_0} Y(t)\{d\hat{A}(t) - dA_0^*(t)\}.$$

Show that $Z(t_0)$ is a mean zero martingale under the null and find the predictable variation process $\langle Z\rangle(t_0)$.

2. Assume that the counting processes $N_i(t), i = 1, \ldots, n$ have intensity processes of the form $\lambda_i(t) = Y_i(t)\alpha_0(t)\exp(\beta x_i)$ where the $Y_i(t)$ are the at risk indicators and x_i are binary taking values 0 or 1. The score test statistic is $U(\beta_0)^\top \mathcal{I}(\beta_0)^{-1} U(\beta_0)$, where $U(\beta)$ are the score equations and $\mathcal{I}(\beta)$ is the expected information: $\int_0^t V(\beta, u)\, dN_\bullet(u) = \sum_{T_j} V(\beta, T_j)$, with U and V defined as in the notes. Show that the score test for the null hypothesis $\beta = 0$ is the log-rank test.

Reading

ABG Chapter 4, sections 4.1.

Cox (1972), and the ensuing discussion (on canvas).